DOCUMENT RESUME

ED 182 148 SE 029 866

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TITLE Some Early Optics: Classical and Medieval. Experiment No. 6.
INSTITUTION Columbia Univ., New York, N.Y., Barnard Coll.
SPONS AGENCY National Science Foundation, Washington, D.C.
PUB DATE Jul 76
GRANT NSF-GZ-2990; NSF-HES-74-17738-A-01
NOTE 48p.; For related documents, see SE 029 865-873; Contains occasional light and broken type

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS College Science; *Color; Higher Education; *Light; *Optics; *Physics; Science Education; Science Equipment; *Science Experiments; *Science History; Scientific Principles; Scientific Research; Scientists

ABSTRACT

Information related to the history of optics with emphasis on the classical and medieval periods is presented. Notes are included on experiments dealing with refraction at a plane interface between two media; refraction by transparent spheres; light, color, and reflection by transparent spheres. (Author/SA)

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Some Early Optics: Classical and Medieval

(Experiment #6)

History of Physics
Barnard College

SE 029 866

SOME EARLY OPTICS: CLASSICAL AND MEDIEVAL

Contents

I. Historical Notes. 1

Background (p. 1); The Rainbow (p. 1); Chronology (p. 6).

II. Experiments. 14

1. Refraction at a Plane Interface between Two Media (p. 14); 2. Refraction by Transparent Spheres (p. 20); 3. Light and Color (p. 21); 4. Reflection and Refraction by Transparent Spheres (p. 27); 5. Further Experiments (p. 31).

III. Notes on Physics. 33

IV. Bibliography. 41

V. Details of Apparatus. 43

National Science Foundation Supported Project
Grant No. HES74-17738 A01
Division of Higher Education in Science

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History of Physics
Laboratory
July 1976

SOME EARLY OPTICS: CLASSICAL AND MEDIEVAL

I. Historical Notes

1. Background:

Insofar as Science is recognizable as a distinct activity -- one to which our present-day Science is the heir and successor -- then the study of light must rank as one of its oldest expressions. Awareness of light, its significance, its subtleties and its mysteries antedate, of course, anything we might characterize as science. Indeed, what we might call a science of light emerged, over the millenia, alongside the emergence of science itself; and its roots, like those of science, reach down into the whole range of human concerns, from the most practical to the most philosophical, from the most material to the spiritual, from the particular to the universal. To trace the history of the science of light, or even the limited aspects that are now connoted by the 'physics' of light, leads us, then, not only into a study of the development of a particular field of inquiry, but also to examine how this became a distinct and separate part of a much broader range of intellectual and practical concerns.

For knowledge of the external world man is indebted to the sense of vision to a far higher degree than to the other senses. Light, as the vehicle of conscious awareness of things outside, is a key element in all probings and practices; as much in techne as episteme, and whether it be in contemplation of the heavens or of the art or the artifacts of the human hands and mind. In the two domains in which classical 'science' excelled, Astronomy and Geometry, light and vision are invariably and ineluctably -- if not always explicitly -- involved. Vision is almost synonymous with perception ("Seeing is believeing!"); and even where perception transcends the palpably or ostensibly physical, the idioms of light and vision pervade ('insight'; 'divine illumination'). Moreover, for light, the boundaries between the external and internal are not easily defined. Vision extends through the eye into (or from!) the brain and mind (or soul).

The earliest steps in scientific development were the recognition of separate, though interconnected, domains in this unbounded territory. One witnesses the gradual emergence of three such, each striving at times for its own autonomy.

First: the propagation of vision, or light, directly or indirectly, between (both to and from!) visible or luminous objects and the eyes -- the essentially mathematical-geometrical features of light -- which formed the basis of the Optics and Catoptrics* of the Greeks or the Perspectiva of the medieval scholars, and latter day 'geometrical optics'. Second: the nature of vision -- the structure and functioning of the eye (of course, the human eye primarily) and the relation of eye and brain -- what more recently has become the subject of physiological and psychological optics. Third: the nature of luminous sources, of color, brightness; and more generally, the physical nature of light and its interaction with essentially non-living matter, of its emission, propagation and absorption.

Before, say, the end of the seventeenth century, it seemed virtually impossible to examine fully any phenomenon associated with light wholly within one of these major divisions. Inevitably, the whole range of issues from emission to perception would be involved. The great treatises on light -- Alhazan's, Witelo's, Kepler's, Descartes' (even Newton's Optics, to some degree) span the whole range of phenomena from the light from the stars, and the colors of the Rainbow, to the structure of the eye and the nature of the optic nerve. Kepler, for example, singles out for painstaking examination (from amongst the innumerable problems of optics and light!) the problems of binocular vision and the refraction of starlight in the atmosphere! Problems of one category -- as we would view them -- seemed to demand answers to those in another. Could one really understand the formation of images, by lenses and mirrors, without a knowledge of the structure of the eye or the function of the pupil, the lens, etc? And could one understand either if the essential nature of light itself -- was it form or substance? -- remained elusive? Were the 'rays' of light, which already featured so prominently in Classical optics, physical realities or merely geometrical abstractions? Although the problems were, in fact, being tackled piecemeal, it seemed as if the whole domain of optics, vision and light had to be resolved in its entirety before any one question could be fully answered!

By the time -- the early eighteenth century -- that physics of light emerged as a more-or-less autonomous enterprise, the subject had a long, diffuse history. Its language and idiom expressed not only established knowledge, but also bore evidence of earlier technique, practices, concepts, conjectures and

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* 'Dioptrics' - a term embracing the treatment of phenomena involving refraction was introduced much later, by Kepler in 1611.

prejudices -- some abandoned, others still firmly entrenched -- and cloaked great areas of ignorance. Later developments sifted and refined this language, and gradually endowed -- if not always consciously -- particular terms with their current, usually more precise and limited meanings -- the ones that we often take tacitly for granted. It is imperative, then, when we turn to the earlier history of the subject, to bear in mind that expressions -- and the practices and techniques with which they are implicitly associated, and which we might regard as simple or unambiguous, for example, ray image, illumination, color, vision -- could not only connote something quite different, but might also conceal some issues of major intellectual concern, usually controversial and often polemical. The need to guard against inappropriate and anachronistic interpretation of 'familiar' terms is not without its reward: it can stimulate a scrutiny of current concepts where familiarity rather than understanding may be responsible for the illusion of simplicity.

And beyond the diffuse, extended 'scientific' implications, there was always -- especially with light -- the still wider spiritual, theological (and at times aesthetic) associations of its vocabulary. For many, in its long history, Experiment, Reason and Divine Revelation were joined together in the task of deciphering the mysteries of light, and the language they used embraced the whole range of their experience.

Our purpose is to illustrate this long and diffuse history by examination of some of the experimental investigations made at different epochs from early classical to late medieval. Such inquiries obviously bring to the forefront technique, method, and interpretation of experiment; but one should not forget that inspiration, motive and goals play an essential role; and that these, for some distant historical epoch and unfamiliar social context, may seem strange and even unscientific to us. But they <u>are</u> part of science. To ask <u>why</u> an experiment was made is often as significant, and revealing, as to ask <u>how</u>.

2. <u>The Rainbow</u>

If we examine, in retrospect of course, what sort of investigations were most successful in advancing knowledge and understanding of light, we are not surprised to find that it is where some limited aspect of the phenomena (in the sense of the three broad divisions above) can be explored and analyzed separately. Such opportunities were inevitably restricted by

the range of phenomena, natural and artificial, available and accessible to experiment in any epoch. Thus the geometrical features associated with direct vision (rectilinear propagation), and reflection (Catoptrics), which are largely independent alike of the source of light and the nature of vision, and which are widely manifest in both nature and artifact, represent the earliest developed aspect of the subject. By contrast, speculation about the nature of primary luminous sources (sun, stars, flames, etc.), although common enough, led to little of enduring scientific significance until very late in the development of the subject (early 19th century). Dioptrics, in which concern was with the passage of light through different media (atmosphere, water, crystal, glass), lagged far behind Catoptrics. Here the power of geometrical considerations was severly reduced by the more elusive (and apparently more subtle) nature of the interaction between light and the medium. And the limited understanding of dioptrics had obvious repercussions in attempts to understand the optics of the eye; not to mention the primitive nature of early anatomical knowledge and the extreme subtlety (still) of the further link between the eye and brain (mind?).

Optics developed, then, in two contexts, in both of which the geometrical features could be separated and were significant: in astronomy, geometry (in the literal sense), and time-keeping; and in connection with mirror-artifacts where the circumstances could be controlled, modified, and extended to advantage. This success, by limitation, had its price: curiosity about some striking and universal aspects of light -- color, for example -- could not be satisfied by the study of phenomena where they seemed sufficiently unimportant to be ignored. Curiosity could not be sated (Where did the key to the mystery lie: in the source of light? In its propagation? In its passage through the media? Or in the eye of the beholder? Or in all of these?), but it was sustained by speculation. And it is no surprise to find speculation most intense, and fertile, where the mysteries of color are found coupled to the regularities of geometry, and both are enhanced by a setting of grandiose nature: in the Rainbow. Here (and to a lesser degree in the halos and mock suns, etc.) was just such a phenomenon. Its appeal was overwhelming. It had been the subject of awe and mystery from times before history, endowed with transcendental meanings alike in pagan myth and sacred religion. To the emerging science of light it

presented a compelling challenge: its simple, circular form encouraged the belief that its meaning would yield to geometry and reason; its striking colors held out the promise of deeper secrets concealed. Completely to comprehend the Rainbow would serve Man better to interpret God's word ("It shall be a token of a covenant between me and the Earth.") and the nature of His handiwork. It was a challenge and an opportunity to Faith and Reason alike.

In our present day science of light and optics the Rainbow occupies such peripheral significance that it needs some effort to realize the prominent role it played in earlier history. (It is an interesting meteorological phenomenon, and an instructive application of established optical principles, but for optics itself little more than a curiosity which often does not even merit mention in a textbook.) From the early middle ages until Newton's time, it is hard to think of a tract on optics that did not include an attempt to solve the rainbow riddle. For most, it comprised the climax of the work -- the final test of understanding, and the consummation, as it were, of all the preceeding efforts -- the synthesis of physics (experiment), mathematics (perspective), natural philosophy (observation) and divine purpose and revelation. And so much of what was achieved and discovered, even by those who did not address themselves explicitly to this subject, was adopted and exploited by the many that did, as to give the illusion, sometimes, that the culmination of all optical exploration was in the elucidation of the rainbow! The Rainbow is an appropriate theme to connect together a variety of illustrative experimental investigations in optics from classical antiquity to the late medieval period.

What one might call the essential physical principles involved in rainbow formation were first grasped at the end of the 13th Century. The interpretation was far from complete, but, in contrast to all earlier attempts (and many later ones), its basic features have been retained -- clarified, extended, and greatly refined, to be sure, but not rejected -- in all later, including contemporary, interpretations. In this sense, this 13th Century theory marks the dividing line between centuries of conjecture and inquiry and the evolution -- albeit delayed by some 300 years -- of the 'modern' science of the rainbow.

The essential ingredients of this first theory were the recognition that the rainbow phenomena represented the aggregate effect of the interaction between the sun's rays and the individual spherical drops of water in the 'cloud', and that in this interaction both refraction and reflection played an ineluctable role in producing the characteristic shape and colors. Yet, astonishingly, at this time the 'correct' geometrical laws of refraction of light in its passage between air and water were not known; knowledge of the relationship between white (sun) light and color, had advanced little (much sophistry notwithstanding) beyond the views current in Aristotle's time; and even the basic cosmic and terrestrial features of the significant geometry of sun, sunlight, cloud and observer were not always properly portrayed or appreciated! (The Greeks, 1500 years earlier, might have done better!) To appreciate both the measure of success, and the limitations, of these medieval achievements, some feeling is necessary for what was known, or believed known, about the reflection and refraction of light, as well as what had been observed and recorded about the rainbow phenomenon itself. It is this context that has prompted the choice of some of the exemplary experiments.

Its role in the evolution of optics apart, the long history of the Rainbow has another important aspect. As a phenomenon of nature, the rainbow lay far beyond (just how far was not always clear!) the range of direct manipulation or control. It was certainly no artifact of man. Yet in large measure it was by studying the properties of artifacts which could be handled and examined at close quarters that the rainbow mystery was, eventually, elucidated. The rainbow was not simply imitated; rather, some of the essential features were replicated artificially, and in this way a phenomenon remote and inaccessible was explored by experiments on a wholly different scale. Here is one of the oldest success stories of experimental science - of understanding gained of one phenomenon by exploration and analysis of another of a wholly different form and scale, but incorporating the same essential principles.

3. Chronology

Below are indicated, in barest outline, some of the most notable contributions to the development of optics by the elucidation of the Rainbow (up to c. 1300 A.D.). (O), (L),

(R) indicate, respectively, concern with Optics (Catoptrics, Dioptrics, Vision); Light (Nature of source, Interaction with matter, Color); and the Rainbow (Observations, Experiments, Interpretations).

Classical Greece (600-300 B.C.)

Ionian (L): Thales (c. 624-565 B.C.), Anaximander (611-547 B.C.), Anaximenes (c. 570 B.C.), Democritus (c. 470-400 B.C.). Speculations on the nature of primary elements. Possiblity of light/vision transmitted through void; particles of light. Incandescent sun; reflection of sunlight by moon.

Western Greek School (O,L): Pythagoras (c. 582 B.C.), Empedocles (c. 490-435 B.C.). Vision as propagation from eye to object. Supremacy of mathematical harmony. Mystique of number.

Aristophanes (O): ("Clouds", 424 B.C.). Early mention of burning spheres and concentration of sun rays on wax tablets.

Plato (L): (427-347 B.C.). "Timaeus": Images/ vision as (mathematical) form/species impression on light. Propagation from eye to object and back to eye ('extravision'). Rectilinear Propagation.

Aristotle (L, R): (384-322 B.C.) Methodology of Science:

> "Knowledge of fact differs from the reason for the fact."

"Meteorology": Observations on Rainbow, form and color; Natural and Artificial Rainbows. Speculations on origin and reflection of sunlight in cloud. Primary and secondary Rainbow. "On Colors": 'simple' colors correspond to (simple) elements -- fire, air, water; black and white to light and darkness. Other colors mixtures of elemental colors:

> "So what is black and shady is mixed with light and the result is red."

"De Anima": Vision 'propagated' (form-species), medium required; analogy with sound.

Alexandrian School (300 B.C. - 200 A.D.)

Euclid (O,L): (330-260 B.C.). Platonic view of vision. Geometrical aspects of Catoptrics only. Laws of Reflection ("Optica").

Aristarchus (of Samos) (O): (c. 310-230 B.C.). Geometry of Solar-Lunar shadows. Astronomical scale of distances.

Appolonius (of Perga): (c. 220 B.C.). Basic geometry of conic sections.

Hero of Alexandria (O): (First Century A.D.). "Dioptra". Practical optics: reflection and refraction. Principle of least time (shortest path).

Cleomedes (O): (First Century A.D.). Observations on refraction of light: Objects in water, setting sun, etc.

*Ptolemy (Claudius) (O): (c. 140 A.D.). Investigation of vision -- physical and mathematical. Optical instruments and experiments; experimental measurement of refraction air/water/glass. Refraction in the atmosphere. 'Law' of refraction: $\hat{i} \propto \hat{r}$. First major work (5 books) on optics -- "Optica"; including binocular vision (partly attributable to Hero?). High point of classical optics.

Galen (of Anazara) (O): (131-201 A.D.). Speculations on binocular vision. Anatomy of eye.

*Alexander (of Aphrodisias) (R): (c. 150-211 A.D.). Elaboration of Aristotelian views. Reality of Rainbow colors. Recorded Rainbow Dark Band.

Arabic-Persian Optical Science (800-1300 A.D.)

Alkindi (L): (801-873 A.D.). Theory (essentially Platonic) of vision.

* "Alhazan" (O,L): (Abu Ali al Hasan, ibn al Hasan, ibn al Haitham, c.965-1039 A.D.). Foremost Arab investigator of optics, and major vehicle for transmission of Classical-Alexandrian optics to Western Europe. Writings translated from Arabic into Latin in Spain/Sicily in latter part of 12th Century. Extensive experimentation and theorizing (expecially geometrical) on Burning Mirrors, Transparent

Spheres, and Refraction. Provided inspiration and basis for much 13th Century European optics, especially Witelo (see below).

"Avicenna" (R, L): (Abu Ali al Husain, ibn Abdalla, ibn Sina, 980-1037 A.D.). Major commentary on Aristotle. Philosophical-Physical-Medical. Structure of eye; function of retina. Theories of vision -- 'intra-' versus 'extra-' vision. Relation between visible and luminous rays. Colors from drops, dew, etc. Emphasis on role of individual drops. Writings on optics, etc. translated from Arabic into Latin in Spain at end of 12th Century.

"Averroes" (L): (Abu al Watid Muhammad, ibn Ahmad, ibn Muhammad, ibn Rushd, 1126-1198 A.D.) Foremost popularizer, interpreter and commentator on Aristotle. ("Physica", "De Caelo et Mundo", "De Anima", etc.) A major influence in introducing Aristotelianism to Western Europe (perpetuating and sanctifying Aristotle's prejudices and misconceptions!) Translated into Latin in early 13th Century.

Qutb al-Din-al-Shirazi: (1236-1311 A.D.). Persian astronomer and physician, (Pupil of Nasir al-Din-al-Tusi (1201-1274 A.D.), one of the greatest of the Arabian-Islamic mathematicians.) Mainly astronomical optics, but also first suggested that the key to the Rainbow colors was to be found in studies with globe of water (cf. Alhazan).

*Kamal al-Din-al-Farasi (L, R): (?-c.1320 A.D.). Persian, pupil of Qutb al-Din. Detailed study and experimental demonstrations of rainbow colors and arcs produced by globe of water in sunbeams. ("God has given us a new way...") Attempted a detailed geometrical interpretation of passage of light twice refracted and once internally reflected (primary bow), and twice internally reflected (secondary bow). Kamal al-Din's account of his experiments and interpretations appear as an extension to a critical commentary on Alhazan's work on burning mirrors and spheres (c. 1300). He explicitly refutes Alhazan's version of Aristotle's theory of rainbow formation -- by reflection only in a concave cloud -- particularly on the basis of observed features of natural rainbows: e.g. rainbow often appears against clear sky on mountains; primary and secondary bows concentric and not related to different clouds; two -- and only two (!) -- rainbows ever seen; no correlation between brightness and the order of colors.

Notice the extraordinary similarity of the discoveries and conclusions of Kamal al-Din and the almost simultaneous

writings of Theodoric, (see below). (Kamal-al-Din's commentary on Alhazan appears to have been unknown in Europe until the 19th Century. Whether or not his ideas percolated, indirectly, into the stream of Western optical science is moot.)

Optical Science in Medieval Western Europe (12th and 13th Centuries)

Adelard of Bath: (12th Century); Alexander Neckam (1157-1217 A.D.); Bartholemew the Englishman (fl. c. 1220-1250 A.D.). Neo-Platonist-Augustinian philosophy-metaphysics of light and vision.

Pierre Abelard (1079-1142 A.D.); William of Moerbecke (c. 1215-1286 A.D.); Alfred the Englishman (c. 1200 A.D.). Early influences of Aristotelian science-philosophy. Growing attention to practice, observation and (primitive) equipment.

Robert Grosseteste (O, L): (1168?-1253 A.D.). Bishop of Lincoln. Intellectual leader of Franciscans at Oxford. Founder of Oxford 'School' of scholastic science-philosophy. Elaborated a methodology which included metaphysics-theology, mathematics, and observation, and stressed the role of experiment. Influences of Neoplatonist, Augustinian, and Aristotelian-Arab writers all blended. Extensive writings on light and color -- much as commentary on Aristotle. Exemplified by a brief discourse on the Rainbow ("De Iride").

> "The study of the Rainbow belongs both to the Student of Optics (perspectiva) and to the physicists: but the fact (quid) is the province of the physicist and the reason for the fact (propter quid) the province of the student of optics."

Stresses the importance of experiment, quotes some experimental 'facts', but does not indicate extensive experimentation of his own. Interprets rainbow in terms of both reflection and refraction -- but in cloud-mist. Sources: Aristotle, Euclid, Hero-Ptolemy, Avicenna, Alkindi, Averroes.

*Roger Bacon (O, L, R): (c. 1214-1292 A.D.). "Dr. Mirabilis", Franciscan at Oxford. Disciple of Robert Grossetest. Set out in an ambitious attempt to construct overall system of arriving at the truth, embracing Doctrine (revealed truth), Reason (including math), and the 'New Experimental Science'.

> "This Experimental Science...wholly unknown to the rank and file of students...alone knows how to test perfectly what can be done by Nature, what by the efforts of Art, what by trickery, what by incantations, conjuration, invocations, deprecations...so that all falsity may be removed and the truth alone, of art and nature, may be retained."

In his most important work "Opus Majus" (c. 1270), an extensive treatment of Light and Optics provide the exemplification of his "Experimental Science', and the interpretation of the Rainbow as the final test and vindication of his optics. Extensive treatment of refraction -- mostly recapitulation. Treatment of color essentially Aristotelian, though with some qualifications. Stresses role of individual drops in formation of rainbow. Gives precise description of primary Geometry of the bow (42°- cone, with the sun-eye as axis).

> "Everywhere (where there are raindrops) there are conditions suitable for the appearance of the bow, but...the bow only appears in raindrops from which there is reflection to the eye."

His conclusions are surprisingly modest and tentative:

> "Reasoning alone does not attest to these matters...I don't think that in this matter I have grasped the whole truth, because I have not made all the experiments necessary."

He concludes with a plea for the further study of the 'new experimental science'. Sources: Mostly as Robert Grosseteste, but also Alhazan's optics.

*Albertus Magnus (L, R): (c. 1206-1280 A.D.). "Dr. Universalis". Leading Dominican at Cologne. Aimed "to make all parts of philosophy [i.e. Aristotle] available to the Latin races" (i.e. Western Christiandom). Critical commentary on Greek, Roman, Jewish and Arabic sources, and, especially, on

Aristotle. Wrote voluminous history of the Rainbow in commentary on Aristotle's "Meteorology": "De Coronis et Iride quae Apparent in Nubibus". Stressed reality of colors (from sprays, prisms, etc., as well as Rainbow), illustrated by already familiar observations and experiments. Theory of color essentially Aristotelian. Sources: Similar to Roger Bacon.

Witelo (O, L): (c. 1230-1300?). Polish/Silesian, probably attached to Papal Court at Witerbo. Author of a treatise ("Perspectiva" or "Optica", c. 1270) on light which was the most influential work of reference until the 17th Century! (Kepler's famous "Foundations of Geometrical Optics" (1604), was subtitled: "A supplement to Witelo's Optics"). A printed version of Witelo appeared in 1533. Witelo's work is based largely on Alhazan's Optics. Extensive practical, experimental and mathematical investigation of 'focusing' mirrors of different shapes, and of transparent spheres. Reproduced, with some spurious extensions, the Ptolemy-Alhazan experimental investigation of refraction of water/glass/air interfaces. Recapitulated most of the familiar facts about the rainbow and color production. Sources: Primarily Alhazan; also Roger Bacon and Albertus Magnus, although not explicitly quoted.

*Theodoric of Freiburg (O, L, R): (c. 1255-1311). Also referred to as 'Meister Deitrich' and (in French) as 'Thierry'. Head ('Provincial') of the Dominican order of Preachers in the German Province. Voluminous writings on Theology, Cosmology, Logic, Natural Philosophy, and Optics, and the most extensive work written (up to that time, c. 1300) on the Rainbow, and Halos, etc. ("On Light and its Origin"; "On Colors"; "De Iride").

The first (at least in Western Europe) elucidation of primary (and secondary) bows in terms of two refractions and one (or two) reflections in individual spherical drops. Experiments performed with water-filled glass spheres in beam of sunlight.

Theodoric's Cosmology and geometry: primitive and essentially incorrect; his interpretation of color: Aristotelian scholasticism.

In his lengthy treatise on the rainbow, etc., he recapitulates many of the well-known facts and observations and comments critically on diverse earlier Greek-Arabic and 13th Century views. He is well aware of the bulk of prior and prevailing knowledge, and the plethora of (inconclusive) theories.

"It is hard on this well-trodden path to find new ways, detours or shortcuts....Nevertheless, I will make a search to clarify that the different hypotheses regarding the colors permit a common understanding."

Theodoric's optics is nothing if not eclectic, but his interpretation of the rainbow is a landmark: nothing better -- and much inferior -- will appear for the next 300 years. His appeal to experiment and respect for the facts of observation (or at least some salient ones), lifted Theodoric's interpretation clear of the impenetrable obscurities of his scholastic logic and misty ambiguities of Aristotelean generalization, to endow it with a validity independent of both.

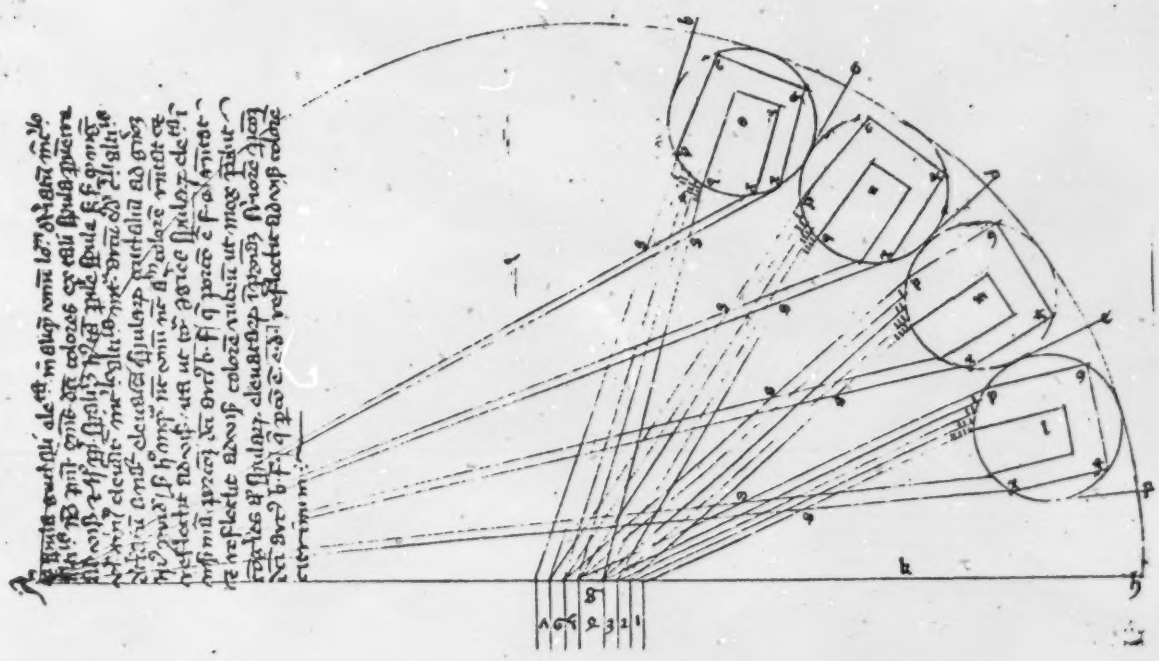


Fig. 15. The formation of the upper rainbow, from Theodoric of Freiberg's *De Iride*, iii. 6–7, MS Basel F. IV. 30, f. 40r. See pp. 257–8. The incident rays should be drawn parallel; the coloured rays emerging from each drop are correctly shown diverging; all the rays of a given colour emerging from the different drops should be drawn parallel. The paths of individual rays inside each drop are not shown.

(Hum. Ref. B.1)

II. Experiments

1. Refraction at a Plane Interface between two Media.

The earliest measurements are those attributed to Claudius Ptolemy, described in Book V of his "Optics" (c. 100A.D.). This work has been transmitted only incompletely and in a far from worthy translation (by Eugenis of Sicily, in the 12th Century), of an earlier Arabic version of the original Greek. But Ptolemy's work certainly inspired the Arabic writers, so that its essential content reached Western Europe by that route also. What is indeed remarkable about these experiments of Ptolemy is that the were, apparently, repeated (or reported), sometimes with additions, but with virtually the same technique, for more than a thousand years! Witelo (late 13th Century) tabulates almost the identical numerical measurements, which were, in turn, based on Alhazan (10-11th Centuries). It seems incredible that although the measurements were quite readily capable of repetition and even of improvement, as late as 1600 the very same data given by Ptolemy still provide the observational basis for Kepler's inquiries! (We can recognize them as identical because of some idiosyncracies of Ptolemy rather than of Light itself!)

Ptolemy's experiments are excellently described in his own account, which are preceded by some general observations. First, there is an assertion that, as in the case of reflection, the incident and refracted ray lie in the same plane as the normal to the refracting surface (which is correct) and, again (by analogy?), that the image of an object observed via refraction appears in the intersection of this normal with the ray which passes from (sic!) the eye to the refracting surface (which although commonly asserted, is not necessarily correct). Secondly, Ptolemy describes the familiar experiment which vividly demonstrated the 'bending' of the visual rays. A coin placed in the bottom of a cup ('baptistir') is cut off from view when the eye is sufficiently low. Water is then poured into the cup and the coin comes in to view. After these preliminaries, an apparatus for systematic measurement and refraction and the results obtained are detailed. (Reproduced, from Cohen & Drabkin on the following pages 15 and 16.)

Ptolemy's account records the results of measurements air → water, air → glass, and water → glass (see tables below) (The arrows indicate the directions of Ptolemy's 'visual ray' which proceeds from the eye to the object observed. This is opposite to the direction we would assign to the passage of light.) Ptolemy also apparently verified -- where this was practicable, as in the case of air and glass -- that the same angular relations applied to vision in both directions: air → glass and glass → air. (But notice that no independent measurements for glass → air are recorded, in which the incident angles, in glass, are changed by uniform steps, of 10°. Might this absence be related to the absence of any attempt to relate the air → glass results to the combined air → water, water → glass measurements?)

With simple apparatus (see pp.43,44), measurements similar to Ptolemy's can be made for air-water, and by straightforward repetition the accuracy of the measurements can be assessed. The results can then be compared with Ptolemy's (and also with calculated values using Snell's Law, and a refractive index air/water of 1.333 -- value for T=20°C). Exact reproduction of Ptolemy's air-glass measurements is not possible as we cannot know the precise optical properties of the glass he used. But one can make similar measurements, test the 'reversibility' of the refraction, and examine the relationship between air → water, water → X, and air → X (Lucite is easier to fabricate than glass). When such a relationship is established, one can ask whether Ptolemy's measurements are in accord with it -- within the accuracy that he might have attained.

Ptolemy's Results

In recording results of our own measurements (and even as do Cohen and Drabkin, in recording Ptolemy's) it is natural enough to use Arabic numerals, and easy to express fractions in decimal notation. But for the Greeks (and Latins), it was quite otherwise. A table or results in Roman numerals looks odd indeed, viz.:

Air	X	XX	XXX	XL	L	LX	LXX	LXXX
Water	VIII	XV et dimid.	XXII et dimid.	XXVIII	XXXV	XL et dimid.	XLV et dimidium	L

The uniform progression: $10°$, $20°$, $30°$...of the chosen angles of incidence presents no problem, but already the recorded refraction angles look clumsy, even for $\frac{1}{2}°$ ('dimidium') precision. Perhaps the manner in which the quantitative relations could be expressed influenced the way in which they were conceived?

But more significant, perhaps, was the very idea of a physical law or regularity or relation which reveals itself (experimentally) in terms of 'arbitrary' and inexact numbers (even if these could be readily expressed). This could hardly have been an appealing idea. Mathematical laws were familiar enough, indeed for Euclid and Ptolemy after him, they were the goal to be sought. But the ideal of mathematics was geometry, and if a physical law, found empirically, were to be related to (or deduced from) a mathematical argument, would not this have to be the sort of exact relationship which geometry could yield? It is not difficult to believe that both the ideal of mathematics or geometry and the particular representation of numerical magnitudes could have influenced Ptolemy in recording his results. After all, he knew that observations could not attain the perfect precision of his ideal mathematics: error were inevitable -- indeed, his tabulated results indicate that approximate values are to expected! How natural then was the temptation to substitute for the actual, fortuitous experimental observations, results which an ideal measurement, representing a true mathematical law, should yield! (The temptation has remained from Ptolemy's day to our's. What has changed is our willingness to record imperfect measurements as well as our conception of what an 'ideal' mathematical law might be.)

When we examine Ptolemy's results, we see just something of this sort. If we represent by $\hat{\delta}$ (the deviation) the difference $\hat{\imath}-\hat{r}$, then in Ptolemy's table we see that the increase in increases with $\hat{\imath}$ in simple, precise, arithmetical progression.

$i°$	10	20	30	40	50	60	70	80
$\delta°$	2	4½	7½	11	15	19½	24½	30
Increment in δ	(2)	2½	3	3½	4	4½	5	5½

One might argue that this uniformity is fortuitous; that given the limited precision of Ptolemy's experiments (an accuracy of $\frac{1}{2}°$ is implicit), this is the way the results turn out. If it were chance that gave to the measurements

of air-water refraction this particularly simple form, then one must assume the same fortuitous circumstance three times: for air-glass and glass-water as well as for air-water!

	$i°$	10	20	30	40	50	60	70	80
Air→ Glass	$\delta°$	3	6½	10½	15	20	25½	31½	38
	Increment in δ	(3)	3½	4	4½	5	5½	6	6½
Water→ Glass	$\delta°$	½	1½	3	5	7½	10½	14	18
	Increment in δ	(½)	1	1½	2	2½	3	3½	4

In all cases the results indicate that the increment in $\hat{\delta}$ increases uniformly with $\hat{i}$, and at the same rate, namely ½° increase for each 10° step in $\hat{i}$! Whatever one may conclude (in the light of one's own measurements) about the relationship between what Ptolemy observed and what he records, of the influence of Ptolemy's record on later investigators of the same subject there can be no doubt.

Alhazan constructed an apparatus very similar to Ptolemy's, with significant refinement in detail, and one essential difference in principle. Ptolemy had studied refraction of the 'visual' ray: Alhazan, using sunlight directly, observed refraction of 'luminous' rays. The identity of the two had by no means been taken for granted -- indeed, the relationship between light and vision was an important topic of investigation from antiquity until well beyond the medieval period. The use of sunlight may also have engendered the more refined sighting technique. Small holes were used in place of colored beads and then were traversed by fine needles -- much in the manner of modern cross-hairs.

Alhazan himself records no actual measurements comparable to Ptolemy's, but Witelo, describing the same instruments as Alhazan, claims to have done so, but he reproduces what seem to be (apart from an odd deviation or two which may be attributed to errors in transcription) the identical figures of Ptolemy, with one major and very telling addition! Witelo misinterprets the reciprocal principal of Ptolemy in the following way. He assumes the 'refraction' deviation, $\delta=(\hat{i}-\hat{r})$, to be equal in magnitude but opposite in direction in the two related measurements: medium a → b and medium b → a. But whereas Ptolemy, correctly, records that this implies that $\hat{r}$ and $\hat{i}$ may be interchanged, Witelo believes that if $\hat{\delta}$ is the deviation in the passage, at incident angle $\hat{i}$ in a, in the passage a → b, then $-\hat{\delta}$ will be

the deviation at the same incidence $\hat{i}$ in b, in the passage b → a! And not only does he imply that he has observed this to be true experimentally, but using the same reciprocal rule, he pretends he has observed the refraction from water to air, for example, for all incident angles, in water up to 80°, with emergent angles in air greater than 90° -- as the application of the false reciprocal rule would imply! (There is in Witelo's record, needless to say, no hint of the actual phenomenon of total internal reflection which takes place at angles greater than the critical angle). This extraordinary disregard for -- or ignorance of -- experimental reality is all the more remarkable in view of Witelo's great skill and wide experience in practical optical work.

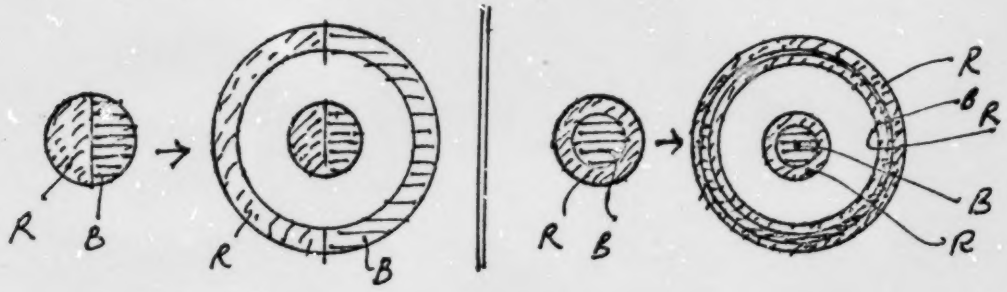
(For references to this section, see pp. 40,41 . For details of Apparatus, see pp. 43,44 .)

2. Refraction by Transparent Spheres

Reported observations and discussions of phenomena which involve refractions at curved interfaces recur in ancient writings in such contexts as burning glasses, the effect of the earth's atmosphere on astronomical observations, the working of the eye, etc. One of the first attempts at systematic optics -- experimental and mathematical -- is Alhazan's, and it deals not surprisingly with the sphere -- mathematically the most appealing and technically the most plausible.

Experiments are performed both by passage of sunlight through transparent spheres (water-filled vessels; spheres of 'crystal') and examination of the emergent light falling on a screen, and by viewing illuminated objects through such spheres. The general qualitative rules and empirical-numerical relationships for refraction -- as recorded by Ptolemy -- sufficed as a basis for a semi-quantitative interpretation of the observations. The analysis is often surprisingly thorough and sophisticated; and the experimental techniques invented to explore the optical phenomena remarkably ingenious. The following delightful example is from Kamal-al-Din-al-Farasi's commentary (c. 1300) on, and extension of, Alhazan's "Optics" (c. 1000). The experiments represent an attempt to verify an earlier analysis (made by tracing, 'theoretically',

individual rays) of the focusing properties of the transparent burning sphere. Here, illuminated patterns of particular shapes (see figure below) are viewed at varying distances from the spheres with both object and eye placed exactly on the same diameter of the sphere.



The images seen, and especially their variation with position of the eye, are most striking. The sudden change in the pattern of images when the eye is moved off axis is particularly noteworthy. Alhazan's and Al-Farasi's interpretations of these phenomena use quite sophisticated arguments and graphical constructions in which the rays of light emerging from a point in the object and falling on the sphere are divided into two groups -- those making small angles with the axis, and those making large angles. In view of the limited (non-analytical) empirical nature of the refraction 'laws' used, their success -- for example in explaining the location of the burning point of the sphere -- is quite remarkable. There is little doubt also that it was from their investigations that Al-Farasi was led to his further investigations of internal reflections in the sphere, and his most significant contribution to interpretation of the Rainbow. (For experimental details, see p. 45)

3. Light and Color

From the innumerable observations of the variegated phenomena of colors, and the endless speculations on their origins and nature, we select a few which illustrate a theme which is elaborated in particular by Theodoric of Freiburg: colors associated with 'radiant' light as distinct from the colors of colored objects; i.e., when the colors appeared associated with the rays of light themselves, and dissociated from the shape or form of any particular object or substance. Naturally enough, the recurrent aspiration was to find one universal principle (or a set of principles) which would

explain all such color phenomena. In retrospect we recognize at least three different types of physical situations in which colors were generated and observed when the primary source was (usually) sunlight.

In the first, sunlight passed through a transparent medium with non-parallel surfaces at entry and exit -- e.g., a hexagonal or triangular crystal (glass) prism or a spherical globe of water. (Refraction was the essential physical process). In the second, the transmission, reflection, or scattering of light involved a medium or surface with spacings or elements of very small size (comparable with the wavelength of light), e.g., the colors of birds' plumage, of light (from particular sources) viewed through fine cloth, or half-closed eyes; or the corona around the sun. (Here the physical basis is diffraction.) In the third class of phenomena, we can (now!) recognize the absorptive or dispersing properties of the medium through which the light passes, or from which it is scattered, as the essential feature; as for example, in the reddening of the sun at sunset, the blue of the sky, or the green of water.

Needless to say, the knowledge of the medieval scholars was far from sufficient to recognize these distinctions, and it was this lack of knowledge that permitted, rather than the extent of understanding that sustained, their elaboration of -- and even confidence in! -- a single unified theory. And so theories did emerge constructed of such abstract generalities which were pliant enough, for all their logical and semantic niceties, to accomodate (a postiori!) almost any observational facts. Nonetheless, the appearance of color was a beckoning sign, suggesting, correctly or not, common underlying causes of diverse phenomenon, stimulating the scrutiny of the phenomena and the search for new ones; and, at times, yielding deeper insights and lasting understanding. It is in this context that one can appreciate the significance of the many observations, and the fewer deliberate experiments that are recorded.

i) Numerous observations are recorded -- and can readily be repeated -- of the occurence of (rainbow) colors at the boundaries of a white illuminated object when viewed through a triangular or hexagonal prism. Similar observations of colors -- projected on walls or screens -- are recorded when shafts of sunlight pass through such prisms. Again it is at the boundaries of dark and light

that the colors are readily observed. Unless there were a ready motive provided by knowledge of the underlying principles -- or one happened by chance -- there would be little reason to explore the color phenomena at great distances from the illuminated object or the prism. On the contrary, with preexisting notions of color as generated by some admixture of black and white (or light and dark), which in one form or another had prevailed from as far back as Aristotle's time at least, it was all too plausible to interpret the observations as fully in accord with such a theory, and to seek no further. But the experiments can, and did, in Theodoric's hands, for example, yield other important results: namely, the clear observation that, in the case of the hexagonal prism, colors appear only when the light enters and emerges from non-parallel sides of the hexagon; and that reflection and transmission alone were not sufficient to generate colors: that refraction involving non-parallel surfaces was essential.

The basic geometrical elements were perceived to involve interfaces between media and changes in direction of the rays, but the clear distinction of roles of place and angle was essentially lacking. The abstraction of a parallel beam of light (or its counterpart in vision), whose lateral dimensions ceased to be significant at large distances, and its realization in practical optical techniques, was far in the future. Beams of light were incidentally defined by the instruments -- prisms, globes, window-apertures -- used to examine their properties. Not surprisingly, color formation was associated with the mode by which light was bounded, and the venerable, and venerated Aristotelian theories of dark and light provided a ready, plausible and accomodating theoretical framework in which to assimilate the facts of observation. For the erudite and intellectually dexterous scholastic the combination of Aristotelean authority and experimental evidence was well-nigh irresitable, and there was no lack of effort, and skill, in weaving the subtle web of arcane theory that would unite the two.

It is instructive to repeat simply, and with no preconceived theoretical notions, the numerous observations of color referred to in the medieval literature; colors produced by shafts on sunlight falling on a screen after passing through prisms or the appearance of luminous or illuminated (white) objects as viewed through a prism, sphere, etc., and to observe how qualitatively plausible

it is to view color as some intermediary of light and dark and produced by their blending, or by the weakening of light by some sort of obstruction.

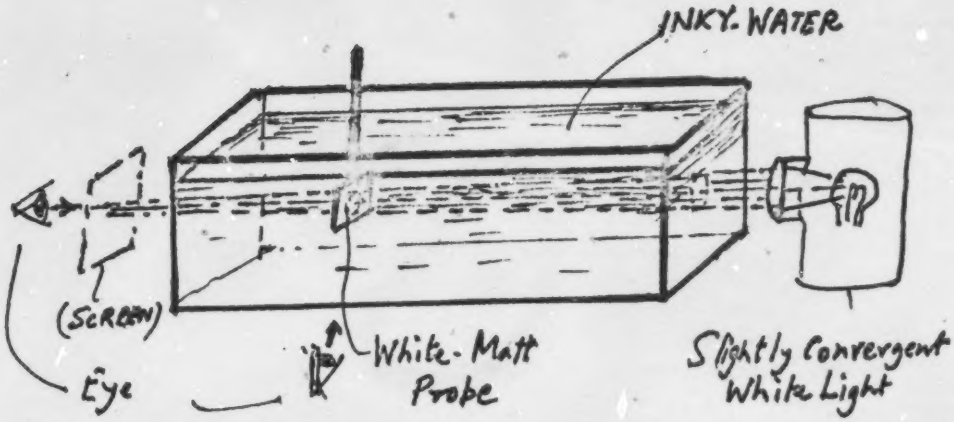
ii) Even more loosely defined -- and therefore more easily assimilated into a preconceived theory -- were the scattered examples of phenomena which we interpret as the production of color by interference, involving objects of appropriately small dimensions or fine structure: e.g. bird's feathers, finely woven cloth, almost closed eyes, etc. But here again, we can recognize a feature which could lead (or mislead) to a unifying interpretation. For interference colors to be seen, it is not enough that the structures be suitably dimensioned; the source of light must also be sufficiently well-defined (a fine 'point' source or a 'parallel-beam', ideally) so that different parts of the light coming from different points in the source, and in different directions, do not produce a multitude of overlapping colored patterns whose combined effect is to wash out all perceptible evidence of color. The natural way to 'sharpen' the quality of light is to introduce defining apertures (one has only to recall the subsequent classic observations of Grimaldi and Young to realize how drastic this reduction of light had to be!), so once again color becomes associated with the bounding or weakening of light. The ancient wisdom that analyzed color in terms of light and darkness seemed again vindicated.

Experimentally, such associations are easily demonstrated. With an extended bright source (open sunlight or an ordinary frosted electric lamp, or a fluorescent light), colors cannot be seen when the light is viewed through some finely-textured material (e.g. fine nylon hose). 'Sharpen' the source of light, by limiting the light with a small aperture, or replacing the large lamp by one with a fine filament (and thereby, incidentally, working at much lower levels of general illumination) and colored 'interference' effects are readily observed. Color, again, is associated with a condition of semi-darkness!

iii) The generation of colors by the selective absorption or scattering in a beam of white light is a more subtle matter. Moreover, the manifestation of color is here

quite different from the characteristic sequence of rainbow colors so often produced by refraction and interference. Nevertheless, the selective processes are here again associated with a reduction in intensity of light, and so the same general principle of blending light and dark could be invoked, albeit now the connection between theory and observation is even more tenuous. The reddening of the sun at sunset is an obvious example; a more dramatic experimental demonstration (mentioned, inter alia, by Albertus Magnus) is the following:

> A small flask or beaker (100 cc.) of water is placed in front of a white-light source or a brightly illuminated white screen. Into the water a few drops of black (drawing) ink are dropped. As the ink diffuses slowly (or is stirred) so as to produce a uniform mixture, the light source, as viewed through the liquid, appears bright red! If a small quantity of the liquid is transferred to a test-tube, and the source viewed through this, the color will be quite different -- orange/yellow or dirty green -- depending on the concentration of ink in the original solution. The whole sequence of color changes in the transmitted light can be observed if a more-or-less parallel (or slightly convergent) beam of white light is directed through a tank with transparent ends (a fish-tank serves well) and filled with the water-ink mixture. Transmitted light can be sampled at various points along the beam by interposing a small disc of white material. In order that the appearance of the disc faithfully represent the 'color' of the beam itself -- at the place where it is located -- the light beam should be arranged just below the water surface and the disc viewed from above. Otherwise the light scattered by the disc is again subject to selective absorption in its lateral propagation, and its color thereby modified.



Needless to say, the precise correlation of these phenomena with one form or another of Aristotelean color theory requires some manoevering! In its original formulation, for example, red is produced by the least, and blue by the most, admixture of black with white. (There are only 3 'simple' colors in all: red, green and blue!) But the particular form of the theory and its tricolor basis in particular, had already been challenged by Theodoric. Arcane variants of the theory were contrived which seemed to reconcile principle and observation -- albeit if these were dealt with one at a time and overall consistency was not too closely scrutinized!

The examples of color phenomena we have discussed, and which were prominent ancient and medieval investigations, were, as already remarked, related to the 'color' of light itself, rather than color of objects. This distinction was clearly felt -- if not clearly formulated or explained -- by the early explorers of the subject. There was much argument about the reality of these so-called 'radiant' colors. For many purposes, we might concisely characterize these two classes of color phenomena as, on the one hand, those where the appearance of color is a function of the angle of the light (or vision) rays rather than their point of origin (refraction, rainbows, halos, diffraction); and , on the other, those where color depends on the location from which the colored light originates and not particularly on angle (the colors in an object or a painted scene).

It is this contrast that underlies many of the perplexing features of the rainbow, and which are resolved only when clearly perceived or formulated. But such a simple distinction could only be made much later in the development of optical science, after Newton had clarified the relationship between white light and color, and between the 'color' of light and the 'color' of illuminated objects.

4. Reflection and Refraction by Transparent Spheres (Rainbows)

It was these investigations, by Theodoric and by Al-Farasi, that provided the most fertile and direct clues in the elucidation of the rainbow puzzle.

They are easily reproduced by use of sunlight; or, if this is impractical, with a little more trouble using an artificial light source. The essential features of this latter -- if the phenomena are to match those observed with sunlight -- are an extended shaft of light, of say 12 cm. or more diameter, all parts of which are characterized by an angular spread of about $\frac{1}{2}^\circ$ (the angle subtended by the sun at the earth). A small arc source (arc size $\lesssim$ 2 mm.) and a parabolic (or spherical) mirror of some 20 cm. focal-length, and aperture radius 6 cm. or more, serves well. A large condensing lens may be used in place of the mirror, but one must take care to avoid confusing effects arising from chromatic aberration.*

i) An ordinary 'spherical' flask (100 cc. capacity, $\sim$3 cm. radius) filled with water is suspended centrally in the light beam, which is -- for simplicity -- horizontal. Various 'images' of the light source are seen in the flask. Some exhibiting some color and complex 'structure' can be seen near the surface. (Additional 'ghost' images may be seen due to reflection at the water/glass interface.) At large distances (several meters) one can, on scanning by eye, pick up a succession of colored images -- more-or-less 'points' of light -- showing rainbow colors. Closest to the outer surface, and viewed at an angle of 42° to the incident light, one can observe a bright red flash. (This corresponds to the red of the primary bow). One can roughly trace the course of the light which produces this flash simply by blocking off parts of the

*Similar effects may be observed with simpler and smaller equipment: e.g., a 'point's source (Tensor lamp) and a smaller globe $\sim$5 cm. diam., with adequate separation ($\gtrsim$ 12 ft.)

incident light with one's finger. (Also, by inserting some obstacle inside the flask, one can verify that the light does not pass through the central part of the globe.) By placing a moist finger on the rear of the globe (opposite the light source) one can locate the points of reflection. It is readily seen that the light from the source enters the globe at one side, is reflected at the back, and then emerges on the opposite side. For each 'ray' there is symmetry about a line bisecting the incident and emergent rays. For the whole assembly of 'rays' comprising the incident light, there is an axis of symmetry passing through the centre of the sphere, parallel to the incident light (see figures). The red flash, etc., may be observed in any direction making 42° with the incident beam. (Note: On account of the finite size of the globe, the proper angle is not obtained by reference to the center of the sphere!)

Standing to one side of the sphere with one's back to the light source (as before) and scanning the opposite side of the sphere, one can pick up similar impressions at an angle of about 53° to the light direction. This corresponds to the secondary bow, with the (angular) sequence of the colors reversed. One can again roughly trace the path of the light rays.

With this arrangement of light and a single sphere, the rainbow colors can be projected on a white surface -- e.g., a white cardboard placed horizontally just below the sphere. One observes colored bands, roughtly parabolic in shape, corresponding to the intersection of the cone of 'rainbow' light with the horizontal plane.

ii) The rainbow-colored arcs may also be seen inside the globe. One hemisphere of the globe is painted opaque white and the bounding circumferance arranged in a vertical plane parallel to the incident light. A large colored band (semi-circle) corresponding to the primary bow, and a much smaller one corresponding to the secondary, can be observed on looking into the sphere from the transparent side. (See figure. p.46)

A procedure which is intermediate between this latter of Al-Farasi, and the former one, of Theodoric, is to

screen off half the incident light by a white card, and view the 'bow' from behind. Or, if a screen with a circular aperture, just a little larger that of the globe, is placed close to it, and between it and the light, the whole circular primary 'bow' will be brightly visible. The secondary bow will not be seen unless the screen is moved away some distance from the sphere. If the white paint is removed from a small region on which the internal 'bows' fall, then the emergent light can be again projected on a screen, and the path of the rainbow light thus vividly demonstrated.

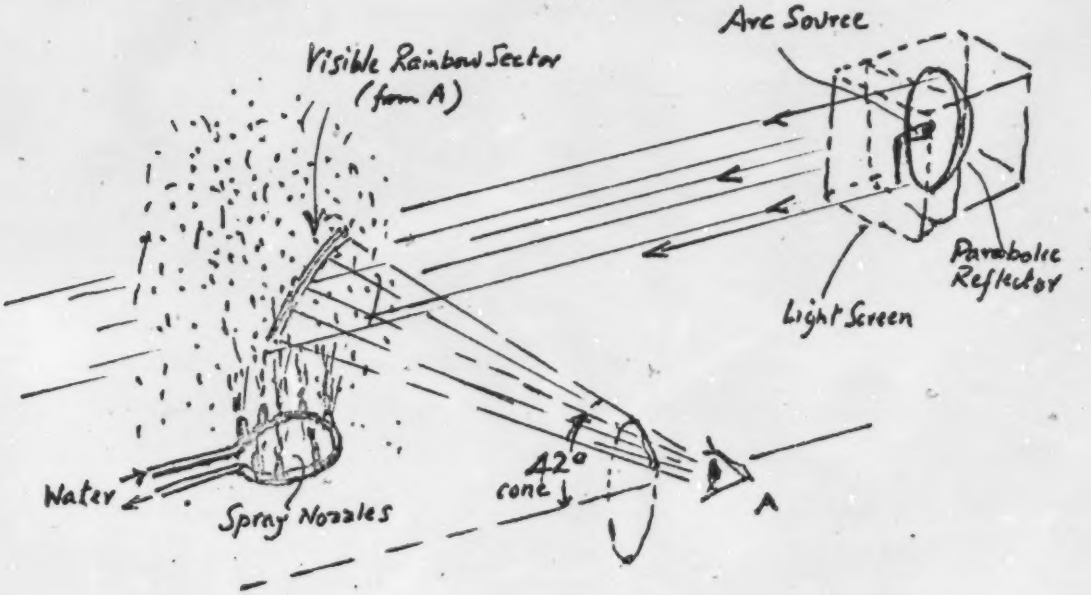
iii) A close simulation of the natural, open-air rainbow is obtained when a shaft of sunlight enters a darkened room and falls on a spray or fountain of water drops. Similar rainbows occur when sunlight is intercepted by the spray from ships' bows, fountains, mill-wheels, waterfalls, etc., and the spray is viewed against a not-too-bright background. Such phenomena had been observed since antiquity and are repeatedly referred to in the literature from Aristotle onwards. Aristotle's own brief description is:

> "A rainbow is also produced when someone sprinkles a fine spray into a room, so placed that it faces the sun and is partly illuminated by it, partly in the shadow...(The rainbow) appears to anyone standing outside at the point where the sun's rays stop and the shadow begins...."

Notice the association, vague perhaps, between light, shadow and color. The position suggested by Aristotle for seeing the rainbow is not difficult to reconcile with some that may conveniently be taken; but any implication that such is necessary is not borne out by observation. Despite numerous repetitions, or repeated references to this 'experiment' by medieval authors, there is little recorded of more thorough or precise examination of its essential geometry.

With a suitable situated and darkened room, sunlight can be brought in at a convenient angle (approximately horizontal; 40° to the horizontal is the limit!) by means

of an adjustable plane mirror (12" square). A suitable spray can be produced by one or more commercially available nozzles (as used in small plant sprays) fed from the tap-water supply.



To see a complete rainbow arc, one must place one's eye close (at a distance of approximately half the dimensions of the illuminated area) to the spray, but some part of it (primary or secondary) can be seen at any convenient distance. With an artificial light source, such as described above (p. 27), the conveniently visible segment of the bow will be smaller, but with a 5" diameter shaft of light, parts of both primary and secondary bows can still be seen simultaneously at distances from the spray of about 2 feet or less.

With simple sighting apparatus (e.g., a meter-stick with protractor and plumb-line) the angular diameter of the bow and the location of its 'axis' can be determined. The apparent 'position' of the bow, its dependence on the position of the observer, and even the change as it is viewed with one or other eye, or both, can be recorded. (Notice that although such artificial bows are in <u>principle</u> identical with natural ones, the location of the water drops which produce them -- not to mention the extent of the illumination -- are different. In the

natural case, both the bow itself and the drops that produce it are (visually) effectively at 'infinity'. But, when viewing an artificial bow at, say, 2 feet, the separation between the eyes is not negligible. With binocular vision, one may adjust one's eyes to 'fix' on the raindrops which will not coincide with a fix on the rainbow!)

Clearly discernible changes in the bow appear with changes of water pressure, spray nozzle, etc., which affect both density and size of drops. In general (if the quality of the light source is adequate), supernumerary bows will be visible at both primary and secondary areas. Visibility of the bows is enhanced, of course, if the light after passing through the spray, falls on a dark, absorbing background.

5. Further Experiments

The experiments previously described are more-or-less replications of historical steps in the elucidation of the rainbow made up to the beginning of the 14th Century. (Anachronistic artifacts, where introduced, are simply for convenience -- incidental to the principles involved.) But an experimental exploration may well evoke questions and suggest inquiries beyond those addressed at a particular historical phase. Limitations in technique, the constraints of contemporary intellectual outlook, or simply lack of motive or fortuitous circumstance may have determined what was or was not explored. But there would surely be no cause similarly to restrain curiosity of the student; one might even wish to stimulate and satisfy it. One or two, of the many possible, extensions of the experiments are therefore indicated.

i) Refraction

Ptolemy wrote regarding the refraction in water:

> "We have not found any perceptible difference in this respect between waters of different densities."

What densities? Hot or cold water? Sea or fresh water? Were the differences sufficient to be observable with

the sort of apparatus that Ptolemy -- or Alhazan -- used? Were they really sought?

These questions can be explored by measurements with water at different temperatures, or with concentrated salt solutions. One might also examine the relationship between density and refraction -- the question which so troubled Kepler in a much later era.

Other fluids -- for example carbon tetracholoride -- might also be measured. Such measurements could also be correlated with subsequent observations with transparent spheres.

ii) Color. Somewhat ironically, Ptolemy mentions the use of colored markers in his refraction experiments. Might he have been able to observe any differences in angles for markers of different colors? This can be tried directly; or a more sensitive test made by using markers partly of one and partly of another color. (A technique used by Newton).

iii) Theodoric and Al-Farasi -- as did later Descartes and Newton -- saw in the experiments with transparent spheres of arbitrarily large size, a complete correspondence with the phenomena ocurring in the much smaller water drops that produced the rainbow. But the phenomena do not correspond completely. With large globes, there is no counterpart to the observed supernumerary bows (an 'interference' effect). Thomas Young (c. 1800) was the first to explain the origin of this difference.

One can observe the production of rainbow colors in single small drops, of, say, 1 mm. diameter, where the structure and color of the rainbow light will display the features corresponding to the supernumerary bows. To examine such small drops -- effectively at infinite distances as one does in viewing the rainbow -- a low-power telescope focused at infinity may be used. A simple spectrometer is quite suitable; it furnishes both telescope and collinator for incident light as well as a scale of angles (See ref. D.3.) With this more refined arrangement, it is also possible to observe many of the

higher order rainbows.

Notice, however, that with such tiny drops one is not likely to observe the detailed path of the rays through different parts of the drop. It was, of course, recognition of just such a connection between the local passage of rays through a sphere and their relationship to the appearances at large distances, that played a significant historical role.

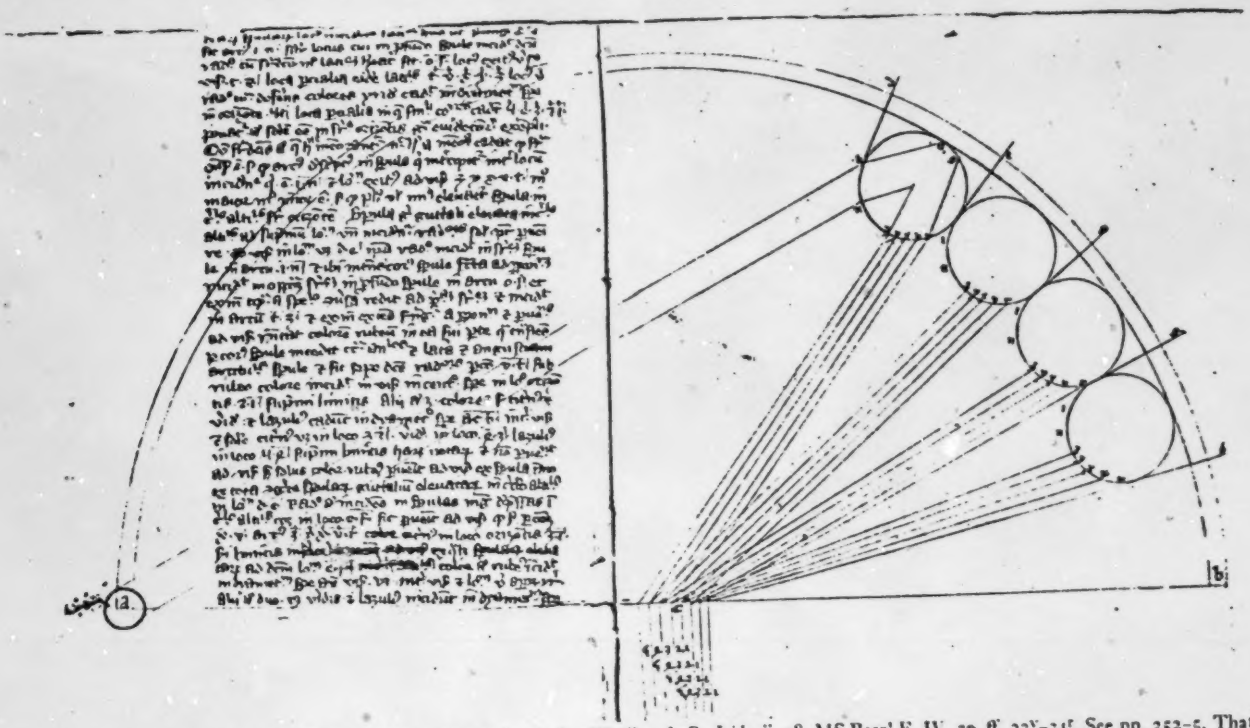


Fig. 13. The formation of the lower rainbow, from Theodoric of Freiberg's *De Iride*, ii. 38, MS Basel F. IV. 30, ff. 33^{v}–34^{r}. See pp. 252–5. That the incident rays going from the sun to the *different* drops should be parallel, which they would not be if they were all shown in the diagram; the coloured rays emerging from individual drop should be diverging instead of parallel; all the rays of a given colour emerging from the different drops should be drawn parallel. The paths of the individual rays inside each drop are not shown (see Fig. 9).

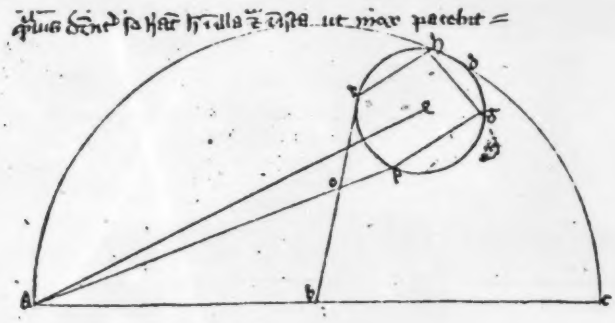


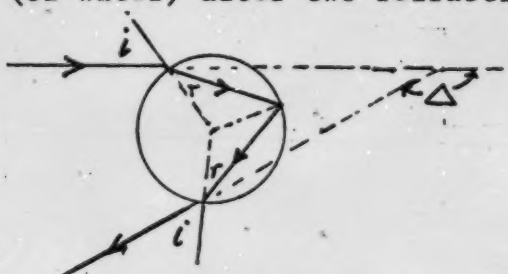
Fig. 14. The determination of the angle between incident and emergent rays producing the upper rainbow, from Theodoric of Freiberg's *De Iride*, iii. 2, 5, MS Basel F. IV. 30, f. 38^{r}. See pp. 255–7.

From Ref. B.1.

IV. Notes on Physics (See ref. D, p. 41)

1. Rainbows

The primary rainbow arc corresponds to the minimum deviation, (the maximum is 180, i.e. reversal of direction of the light), of any ray of light - treated in geometrical-optical approximation - reflected from a spherical drop (of water) after two refractions and one internal reflection



From the symmetry it is readily clear that the deviation, $\Delta = \pi + 2i - 4r$

i.e. $\pi - \Delta = 2(2r - i)$, (Primary)

This yields a minimum Δ at incident angle i_1:

$\cos i_1 = [\frac{1}{3}(\mu^2 - 1)]^{1/2}$,

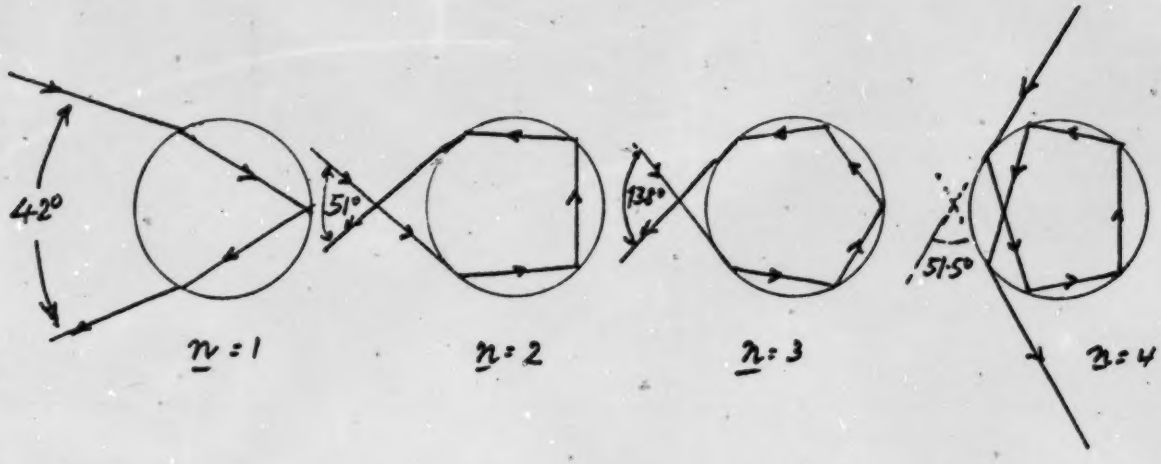
with μ the refractive index for the appropriate 'color' of light. For higher order rainbows, with n internal reflections the corresponding relations are:

$$\Delta = n\pi + 2i - 2r(n+1), \quad \text{and} \quad \cos i_n = [(\mu^2 - 1)/n(n+2)]^{1/2}$$

These relations define the so-called 'Cartesian' rays and angles, which are determined by simple geometrical optics alone. Different component colors (wavelengths) comprising the incident white light have slightly different Cartesian angles. (See table), (This interpretation of the colors is Newton's, not Descarte's.)

The Cartesian analysis, with appropriate values of μ yields the approximate positions of the brightest parts of the rainbow. The actual brightness is determined by two factors: i) the Cartesian concentration of rays, ii) the fraction of the light which is tran
cidence and emergence, and which is reflected at the one, or more, internal reflections. This factor can be estimated by using the Fresnel relations. It is by no

means a maximum for the Cartesian rays, but for an incident angle much closer to 90° .



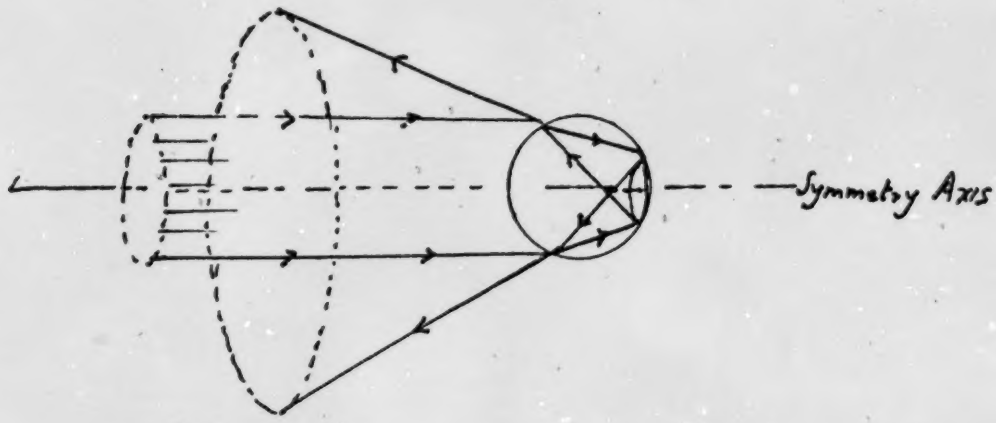
Cartesian rays for water drop, n = 1, 2, 3, 4
(Angles correspond to yellow light, μ = 1.333)

Table: Characteristics of Primary & Secondary Bows

Cartesian Ray		n = 1	n = 2
Angle of incidence/ emergence, $\sin i = \frac{n}{a}\sin r$	Red (μ = 1.331)	59°31′ (0.862)	71°54′ (0.951)
	Blue (μ = 1.345)	58°48′ (0.855)	71°30′ (0.948)
Deviation (Δ)	Red	$\pi - 42°22'$	$\pi + 53°36'$
	Blue	$\pi - 40°36'$	$\pi + 50°24'$

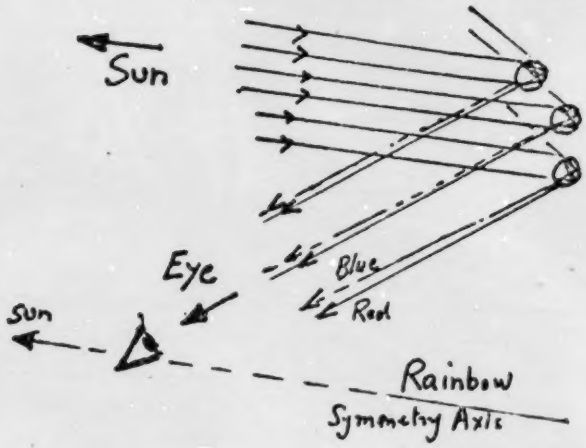
Quite generally, the Cartesian rays approach closer to the surface of the sphere (i approaches 90°) as the order n increases. Notice that the light path is symmetrical

about a line bisecting incident and emergent rays, for any ray, Cartesian or otherwise, in a particular plane. But for a whole collection of <u>parallel</u> rays (sunlight), the overall symmetry is about the direction of this incident light. (Pattern obtained by rotating pattern of individual rays about this direction.)



For parallel light falling on a single sphere, the cone of rainbow light is unique. Thus, rainbow-like figures (circles, parabolas, etc.) may be projected on a (plane) screen which intercepts the cone, but a rainbow does not appear to the eye directly, since this selects a particular direction.

For an extended array of spheres (a raincloud), there is a rainbow cone appropriate to each sphere. Any rainbow-light reaching the eye must also be part of a rainbow-cone, and therefore make the proper angle (42) with the sunlight. Thus, any drop which lies on a cone of this same angle, with its axis through the eye and parallel to the sunlight, will be visible by rainbow-light, i.e., it will appear bright (and colored, on account of the different Cartesian angles for different colors). Thus each observer sees a different rainbow (the rainbow light from different drops), and for each there is an individual axis of symmetry - through the observer's eye and parallel to the sunlight.



The observer's eye selects the particular light that forms the bow. Such a rainbow, from an extended array of drops, cannot be projected on a screen.

Notice the alternatives - a single sphere: no visible bow, but projection possible;

- a collection of spheres: the bow (or part of it) visible; projection impossible.

Not all the earlier writers on the rainbow clearly perceived the relationship between the appearances with the single sphere and the spray of drops. Some even claimed that a spray-produced bow could be projected on a screen, thereby betraying a reliance on hearsay rather than their own observations!

In all the above considerations, the wavelength of the light, and therefore the diameter of the spheres, are not explicitly involved - there is no linear scale of dimensions. Color enters purely phenomenologically via chromatic dispersion. In actual rainbows the drop sizes are such that diffraction effects are significant, and need to be taken into account to explain what is observed - in particular the supernumeray bows. (The significance of diffraction was recognized by Thomas Young; the first detailed calculations were made by Airy.) The diffraction effects are greater than might at first sight be imagined. This is because the incident light is concentrated by the drop into regions of much smaller dimensions than the drop itself. Thus for a drop of radius 1 mm., the angular spacing of the diffraction maxima and minima are of the same order as the refractive dispersion between, say, read and yellow, i.e. $\sim\frac{1}{2}^\circ$ or 10^{-2} radians. (Notice, however, that is much smaller, 6×10^{-4} rad.)

2. Transmission through a Refracting Sphere (cf. p. 20)

As an optical instrument or element, the transparent sphere is not, today, of much significance (except, perhaps, to fortune tellers). Historically, it (or the hemisphere) was the first object with curved surfaces to be studied. Apart from the appeal of its perfection, it was also used at times (e.g., by Roger Bacon) as a crude

representation of the eye. In the 19th Century, Fresnel found it expdient to use small liquid drops to concentrate sunlight into an effective point source!

To relate the optics of the sphere to more modern optics, it is natural enough to consider primarily the properties of the paraxial rays. This restriction assumed its significance only after the importance of paraxial rays had been emphasized by the extensive development of optical instruments - the telescope in particular. It was first clearly expressed by Kepler in his 'Dioptrics' of 1611. But already in the work of Alhazan and others, the distinction between the paraxial and the widely diverging rays is made ad-hoc, and leads to a clarification of the observed phenomena (ref.C1, p.42)

With the conventional paraxial theory for the passage of light through spherical interfaces, of radius R, between media of refractive index μ, μ' we have as the relationship between object and image positions, S, S':

$$\frac{\mu}{S} + \frac{\mu'}{S'} = \frac{\mu' - \mu}{R}$$

With application of this relation to both entry and exit of the light, we get for the distance of the focus ('burning point') from the center of a sphere of refractive index μ, in air (whose assumed refractive index is unity):

$$d = R \, \frac{\mu}{2(\mu - 1)}$$

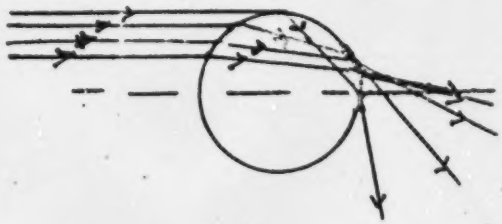
(Water : $\mu = 1.33$; $d = 2R$; for 'typical' glass $\mu = 1.5$, $d = 1.5R$)

These results may be readily verified. Also, by screening the incident light, one may check that it is only the paraxial rays ($2i \lesssim 30^\circ$) that contribute to the

bright focal spot.

For parallel rays, or distant objects/light-sources, it is clear that the central part of the sphere acts like a thick bi-convex lens, and produces a real 'upright' image, which can be clearly seen if the eye is placed sufficiently far away from the sphere.

But the rays incident at $i \gtrsim 30°$, whether parallel, or those diverging from a distant point, do not intercept the axis at a common point (no focus). Moreover, these emerge from the sphere highly divergent; few reach the eye unless this is placed close to the sphere. Concentrations of these outer rays are reponsible for the surprising appearance of the objects viewed by Alhazan/Al-Farasi. It is instructive to vies wthese objects at different distances and with the light to the central region, or to the outer annulus, of the sphere blocked off.



Total Deviation: $2(i-r)$

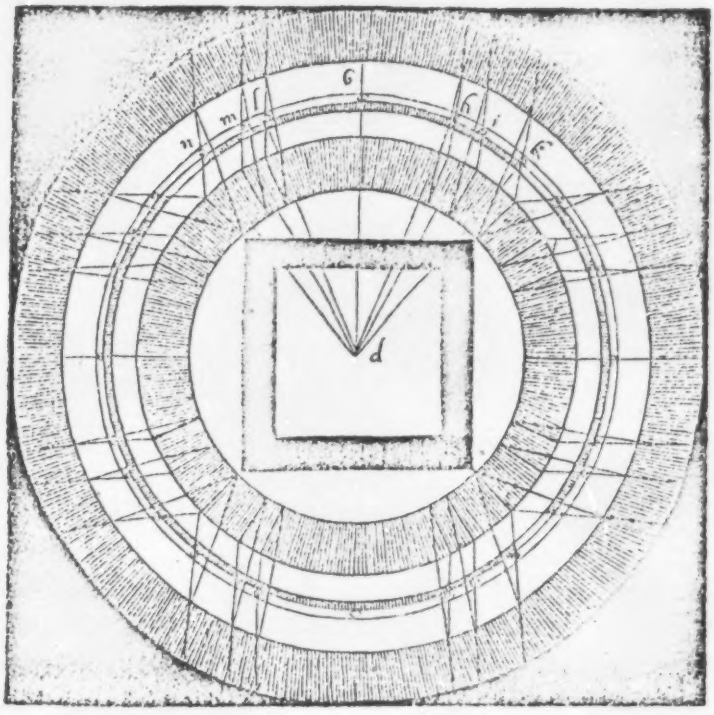
3. Color and Absorption (cf. p.24)

The remarkable color effects observed in the passage of light through a medium containing dispersed scattering/absorption particles arises from the dependence of this scattering/absorption on wavelength. (This problem was first examined by Rayleigh in connection with the blue color of the sky). Whatever the precise nature of the ink-particles, and their interaction with light, it seems that, typically, the scattering increases as the wavelength decreases. The water itself, for the path-length used, can be seen to have little effect on the color of the light.) A $1/\lambda^4$ dependence - which is applicable to scattering by the molecules of the air - implies that the 'resonant' frequencies of the scatterer lie at much higher frequencies than the incident light. In any event, the experiments indicate that in passage through the ink suspension, the shorter wavelength components of the originally white light are successively stripped away so that finally - provided the path-length

is sufficient - light of deep red color emerges.

As the process develops along the light path, the residual light changes from greenish to yellow to orange to red. Its color can be observed in the light scattered from a small matt-white object inserted in the beam. The light scattered from the suspension itself, at any point, depends on two factors: the residual color-components in the beam at that point and the preferential scattering of shorter wavelengths.

It is a remarkable tribute to the resourcefulness of scholastic theories of color that the explanation of the colors appearing here and in prismatic refraction could be subsumed under the same principles!



Alhazan's illustration of Refraction

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Apparatus

1. Refraction at Air/Water interface (pp. 43,44)

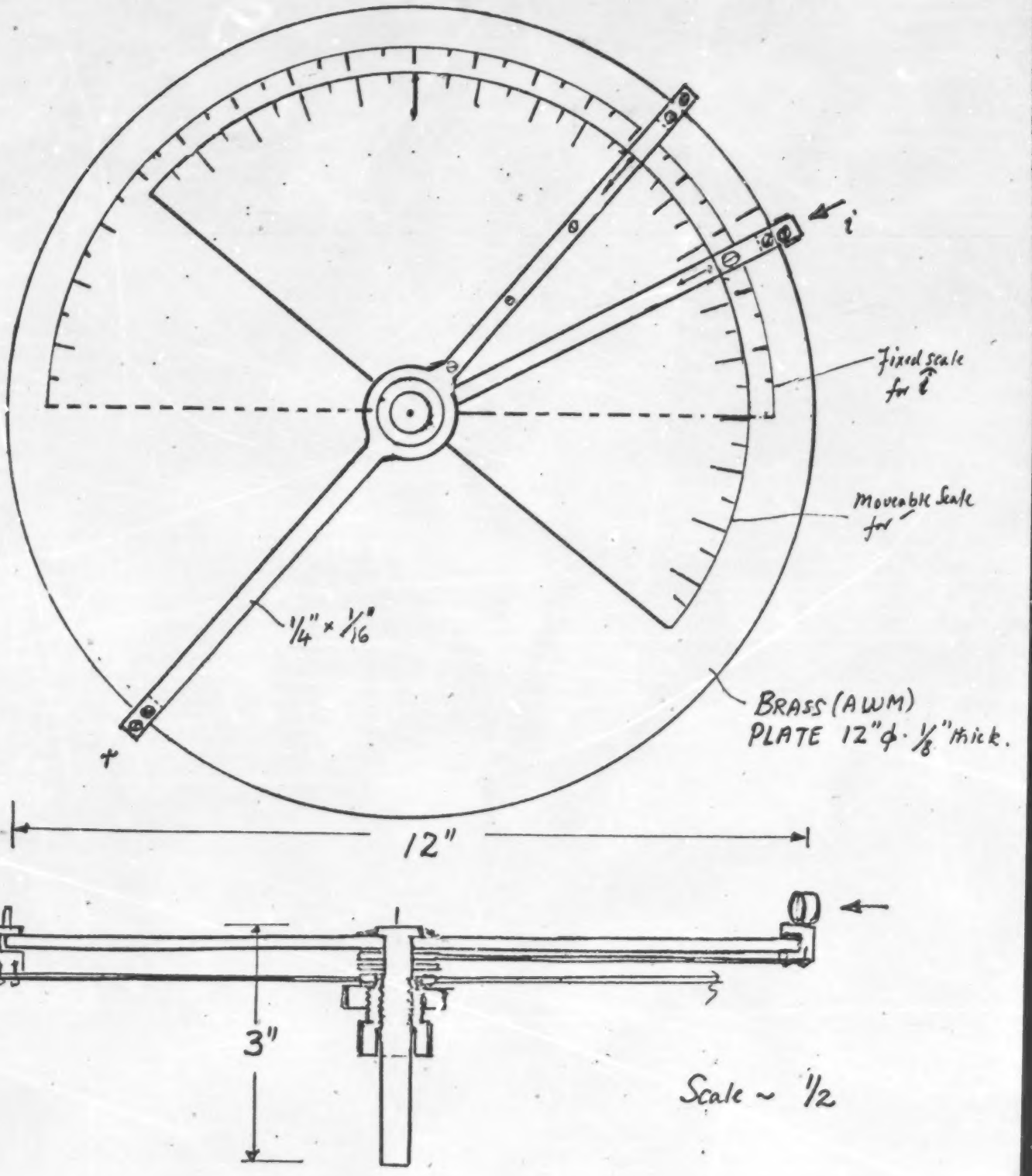
   Rough simulation of Ptolemy/Alhazan apparatus. As shown, arranged for 'visual rays', i.e. viewing submerged object from outside water. By replacing central pin by a screen with small aperture, and with suitable screens to channel light along the 'i'-arm, it can also be tested with 'luminous' rays. (Of course, many much simpler arrangements can be used to measure the refractive index of water - at least approximately. The present apparatus is sufficiently close to that used by Ptolemy - Alhazan to examine, e.g., whether the observations, as recorded, were influenced by preconceived notions of mathematical regularity.)

2. Refraction through Sphere. Alhazan, Al-Farasi, etc. (p. 45)

   The arrangement permits the spacing between illuminated object, refracting sphere and the eye to be varied, while maintaining the object-eye line colinear with the center of the sphere. With sufficient illumination, images can also be projected on a screen. The flask is of sufficient size to transmit widely divergent, as well as paraxial, rays.

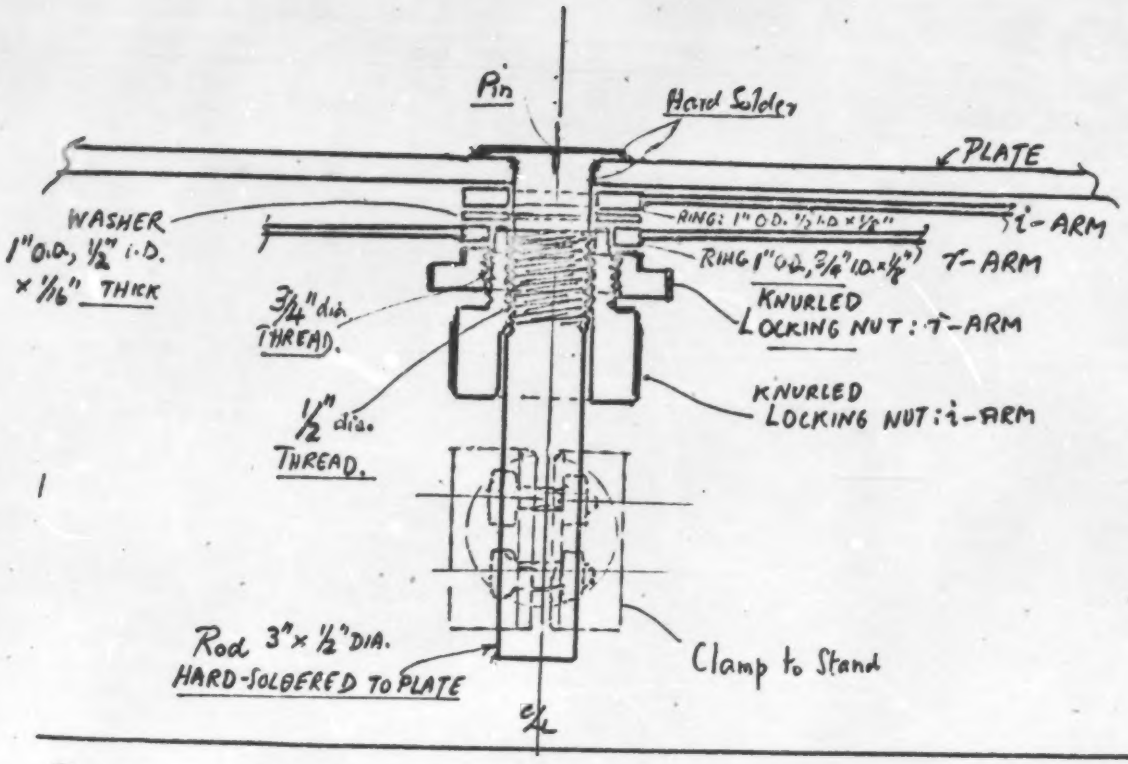
3. Reflection and Refraction in Sphere. (p. 46)

   Arrangements similar to those described by Al-Farasi for observing the 'rainbow': (a) 'painted' on inside of sphere, (b) projected on screen outside. In (b) a complete circle may be observed, with the screen perpendicular to the incident light. With a screen parallel to the light, the projected bow is parabolic.

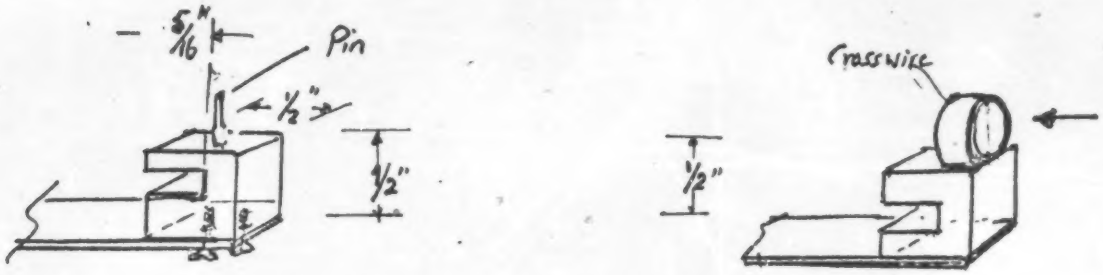


Refraction Apparatus ('Alhazan')

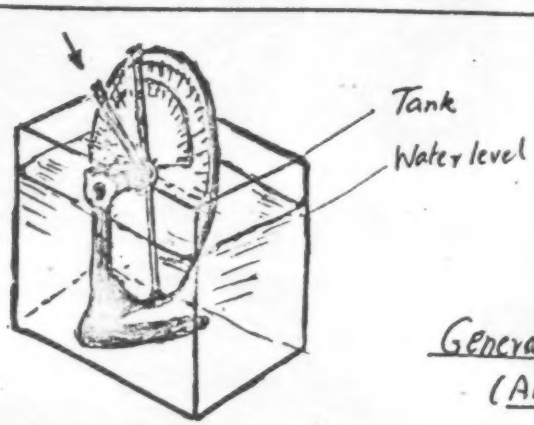
7/76
SD



Details of Locking and Mounting. Scale 1/1



Details of 'Object' and 'Eyepiece' Scale 1/1



General Arrangement
(Air-Water)

Refraction Apparatus: Details.

7/76
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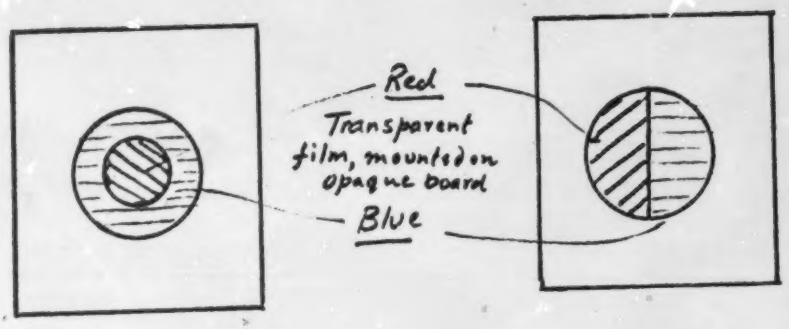
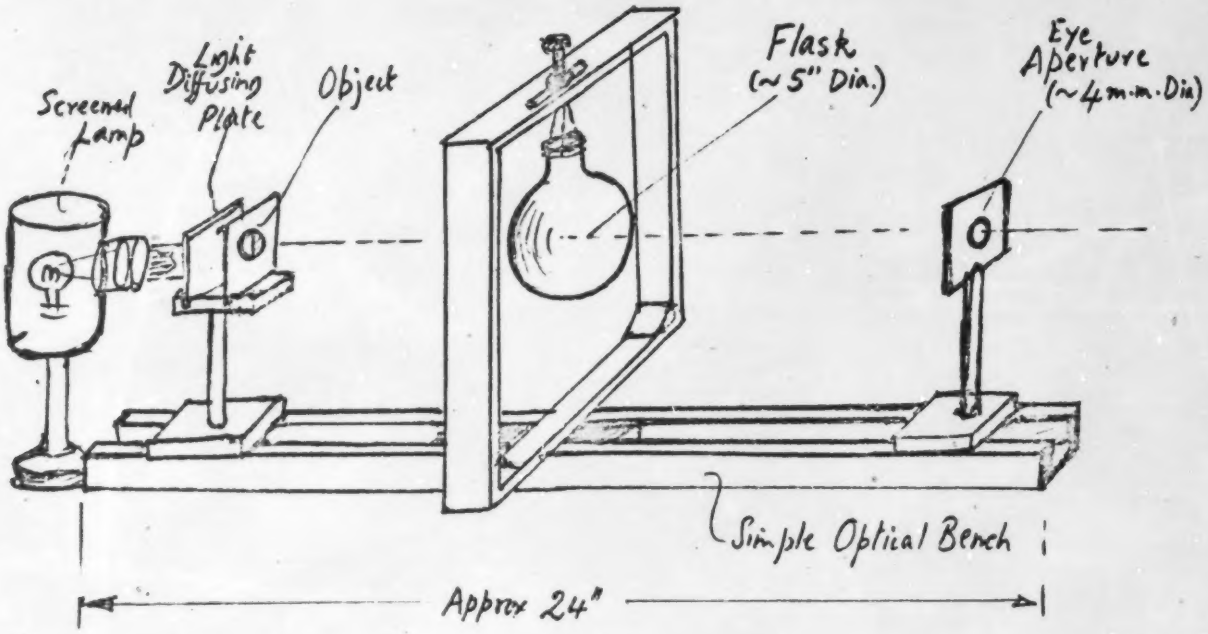
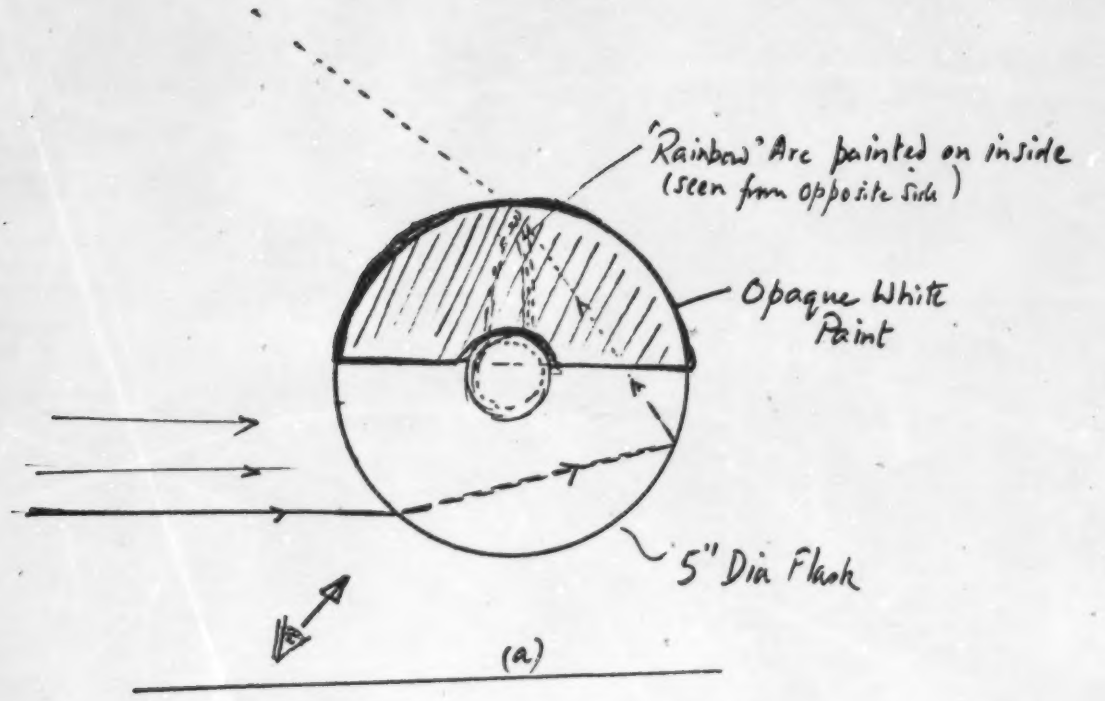
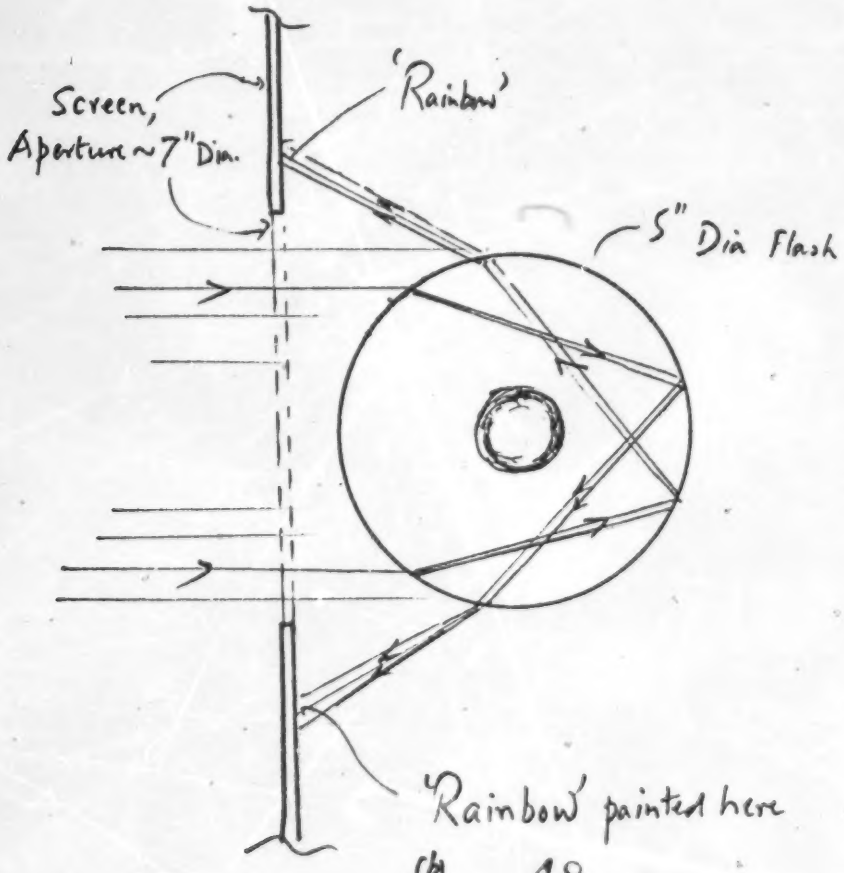




(Simple Arrangements for transverse adjusments necessary)

TRANSMISSION THROUGH SPHERE (ALHAZAN, AL-FARASI)





48

Internally & Externally projected 'Rainbows'. - Al-Farasi

7/76
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